Errata

Errata and Addenda to "Application of Oscillatory Aerodynamic Theory to Estimation of Dynamic Stability Derivatives"

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THE pitching moment coefficients in Ref. 1 are referred to an axis through the wing apex with the pitch axis through the quarter-chord point of the mean aerodynamic chord (MAC). It was the authors' intention to present the coefficients using the same reference and pitch axes at the quarter-chord point of the MAC. The example wing had an aspect ratio of 8.0, a taper ratio of 0.25, a quarter-chord sweep of 30 deg., and was flying at a Mach number of 0.8. Using a new lattice idealization of the wing (since the original idealization was not recorded) defined by 5 equal chordwise divisions and 15 variable spanwise (narrower toward the tip) divisions, the new lift and corrected moment coefficients for the quarter-chord MAC reference axis and pitch axis are presented in Table 1 for the nine values of reduced frequency previously considered. The calculations were based on the doublet-lattice method (DLM) of Ref. 2 as integrated into NASTRAN®[3] and were carried out using MSC/NASTRAN; the original calculations were based on the DLM of Ref. 4 which was later developed into Ref. 5. Slight differences in the lift coefficients between the present and earlier solutions can be attributed to a different lattice idealization.

If we extend Ref. 1 to permit different reference chords for reduced frequency ($k=\omega b/V$) and pitching moment coefficient

$$C_m = M/\tfrac{1}{2}\rho V^2 S\bar{c}$$

since aeroelasticians and aerodynamicists prefer different reference geometry, and only consider the lower order stability derivatives, we may rewrite and truncate Eqs. (12-15) of Ref. 1 to read

$$\bar{C}_z/\alpha_0 = C_{z_\alpha} + i(k\bar{c}/2b)\,(C_{z_{\dot\alpha}} + C_{z_q}) \tag{1}$$

$$\bar{C}_m/\alpha_0 = C_{m_\alpha} + i(k\bar{c}/2b)\,(C_{m_{\dot\alpha}} + C_{m_q}) \tag{2}$$

$$\bar{C}_z/h_0(2/\bar{c}) = i(k\bar{c}/2b)\,C_{z_\alpha} - (k\bar{c}/2b)^2 C_{z_{\dot\alpha}} \tag{3}$$

$$\bar{C}_m/h_0(2/\bar{c}) = i(k\bar{c}/2b)\,C_{m_\alpha} - (k\bar{c}/2b)^2 C_{m_{\dot\alpha}} \tag{4}$$

from which

$$C_{z_\alpha} = Re(\bar{C}_z/\alpha_0) = (2b/\bar{c}k)\,\mathrm{Im}[\bar{C}_z/h_0(2/\bar{c})] \tag{5}$$

$$C_{m_\alpha} = Re(\bar{C}_m/\alpha_0) = (2b/\bar{c}k)\,\mathrm{Im}[\bar{C}_m/h_0(2/\bar{c})] \tag{6}$$

$$C_{z_{\dot\alpha}} = -(2b/\bar{c}k)^2\,\mathrm{Re}[\bar{C}_z/h_0(2/\bar{c})] \tag{7}$$

$$C_{m_{\dot\alpha}} = -(2b/\bar{c}k)^2\,\mathrm{Re}[\bar{C}_m/h_0(2/\bar{c})] \tag{8}$$

$$C_{z_{\dot\alpha}} + C_{z_q} = (2b/\bar{c}k)\,\mathrm{Im}[\bar{C}_z/\alpha_0)] \tag{9}$$

$$C_{m_{\dot\alpha}} + C_{m_q} = (2b/\bar{c}k)\,\mathrm{Im}[\bar{C}_m/\alpha_0] \tag{10}$$

$$C_{z_q} = (C_{z_{\dot\alpha}} + C_{z_q}) - C_{z_{\dot\alpha}} \tag{11}$$

$$C_{m_q} = (C_{m_{\dot\alpha}} + C_{m_q}) - C_{m_{\dot\alpha}} \tag{12}$$

Table 2 presents the derivatives calculated from Table 1 and Eqs. (5-12). It was mentioned in Ref. 1 that C_{z_α}, C_{m_α}, C_{z_q}, and C_{m_q} could be calculated as above but that they are static derivatives and should be calculated as such. Static values determined by the vortex-lattice method§ for the new lattice idealization are $C_{z_\alpha} = -5.856$, $C_{m_\alpha} = -0.5653$, $C_{z_q} = -5.947$, and $C_{m_q} = -3.206$.¶ These can be compared to the vaues estimated by the oscillatory theory in Table 2.

The transformations from one moment reference axis to another are well known, but the transformations from one pitch axis to another are not. Let x_1 and x_2 be the first and second moment reference axes, respectively, such that $x_2 = x_1 + \Delta x_1$ (x is positive forward, as in Ref. 1), and x_3 and x_4 be the first and second pitch axes, respectively, such that $x_4 = x_3 + \Delta x_3$. Then the moment coefficient about the second moment axis $C_{m_{\alpha_2}}$ is found from the coefficient about the first axis $C_{m_{\alpha_1}}$ as usual,

$$C_{m_{\alpha_2}} = C_{m_{\alpha_1}} + C_{z_\alpha}(\Delta x_1/\bar{c}) \tag{13}$$

Since the $\dot\alpha$ derivatives are found from a pure plunging motion, the transformation of Eq. (13) also relates $C_{m_{\dot\alpha_2}}$ to $C_{m_{\dot\alpha_1}}$ so we have

$$C_{m_{\dot\alpha_2}} = C_{m_{\dot\alpha_1}} + C_{z_{\dot\alpha}}(\Delta x_1/\bar{c}) \tag{14}$$

In terms of the static aerodynamic influence coefficients (AIC's) of Ref. 6 $[C_{h_s}]$ (also discussed in Ref. 1) the lift coefficent for pitching about the first pitch axis is

$$C_{z_{q_3}} = (1/\bar{c}^2)\,\lfloor I \rfloor\,[C_{h_s}]\{x_3 - x)^2\} \tag{15}$$

For pitching about the second pitch axis it becomes**

$$C_{z_{q_4}} = (1/\bar{c}^2)\,\lfloor I \rfloor\,[C_{h_s}]\{x_4 - x)^2\}$$
$$= C_{z_{q_3}} + 2C_{z_\alpha}(\Delta x_3/\bar{c}) \tag{16}$$

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§The static values require a DMAP ALTER to obtain them from current versions of NASTRAN. A static aeroelastic capability is presently being added to MSC/NASTRAN.

¶The footnote on p. 273 of Ref. 1 should be corrected for the calculation of the pitch rate derivatives: the quadratic deflection mode for an effective parabolic camber is $\{h\} = \{(q/2V)(x_1 - x)^2\}$, so for a unit value of $q\bar{c}/2V$, the deflection mode is $\{h\} = (1/\bar{c})\{(x_1 - x)^2\}$.

**Note that there is no static lift from plunging, i.e., $\lfloor I \rfloor [C_{h_s}]\{I\} = 0$.

Table 1 Lift and moment coefficients for pitching and plunging of a typical jet transport wing at a Mach number of 0.8

k	Pitching $\bar{C}_z/\alpha_0$	Pitching $\bar{C}_m/\alpha_0$	Plunging $\bar{C}_z/ikh_o(2/\bar{c})$	Plunging $\bar{C}_m/ikh_0(2/\bar{c})$
0.001	$-5.856+i0.0067$	$-0.5653-i0.0023$	$-5.856+i0.0126$	$-0.5653+i0.0009$
0.002	$-5.856+i0.0134$	$-0.5652-i0.0046$	$-5.856+i0.0252$	$-0.5652+i0.0018$
0.004	$-5.855+i0.0267$	$-0.5651-i0.0093$	$-5.855+i0.0505$	$-0.5651+i0.0035$
0.005	$-5.855+i0.0333$	$-0.5650-i0.0116$	$-5.854+i0.0630$	$-0.5650+i0.0044$
0.010	$-5.849+i0.0660$	$-0.5643-i0.0233$	$-5.847+i0.1254$	$-0.5642+i0.0087$
0.020	$-5.826+i0.1278$	$-0.5615+i0.0473$	$-5.820+i0.2459$	$-0.5612+i0.0168$
0.040	$-5.749+i0.2288$	$-0.5524-i0.0981$	$-5.724+i0.4609$	$-0.5514+i0.0297$
0.050	$-5.698+i0.2662$	$-0.5466-i0.1249$	$-5.662+i0.5529$	$-0.5452+i0.0346$
0.100	$-5.398+i0.3251$	$-0.5123-i0.2730$	$-5.278+i0.8558$	$-0.5089+i0.0433$

Table 2 Estimates of $C_{z_\alpha}, C_{m_\alpha}, C_{z_{\dot\alpha}}, C_{m_{\dot\alpha}}, C_{z_q}$ and C_{m_q} based on a single value of k

k	C_{z_α}	C_{m_α}	$C_{z_{\dot\alpha}}$	$C_{m\dot\alpha}$	C_{z_q}	C_{m_q}
0.0	−5.856	−0.5653	–	–	−5.947	−3.206
0.001	−5.856	−0.5653	12.63	0.8883	−5.947	−3.206
0.002	−5.856	−0.5652	12.63	0.8878	−5.946	−3.206
0.004	−5.855	−0.5651	12.62	0.8861	−5.945	−3.206
0.005	−5.855	−0.5650	12.61	0.8848	−5.944	−3.206
0.010	−5.849	−0.5643	12.54	0.8744	−5.936	−3.205
0.020	−5.826	−0.5615	12.30	0.8388	−5.907	−3.203
0.040	−5.749	−0.5524	11.52	0.7432	−5.803	−3.195
0.050	−5.698	−0.5466	11.06	0.6916	−5.734	−3.190
0.100	−5.398	−0.5123	8.56	0.4326	−5.307	−3.162

$$C_{m_{q_{1,3}}}=(1/\bar{c}^3)\lfloor x_1-x\rfloor [C_{h_s}]\{(x_3-x)^2\} \tag{17}$$

The moment about the first moment axis for pitching about the second pitch axis is

$$C_{m_{q_{1,4}}}=(1/\bar{c}^3)\lfloor x_1-x\rfloor [C_{h_s}]\{(x_4-x)^2\}$$
$$=C_{m_{q_{1,3}}}+2C_{m_{\alpha_1}}(\Delta x_3/\bar{c}) \tag{18}$$

The moment about the second moment axis for pitching about the first pitch axis is obviously

$$C_{m_{q_{2,3}}}=C_{m_{q_{1,3}}}+C_{z_{q_3}}(\Delta x_1/\bar{c}) \tag{19}$$

Finally, the moment about the second moment axis for pitching about the second pitch axis is

$$C_{m_{q_{2,4}}}=(1/\bar{c}^3)\lfloor x_2-x\rfloor [C_{h_s}]\{(x_4-x)^2\}$$
$$=C_{m_{q_{2,3}}}+2C_{m_{\alpha_2}}(\Delta x_3/\bar{c}) \tag{20}$$

Equations (14), (16), (18), and (20) are variously found in the USAF DATCOM.[7]

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Aeroelastic Divergence of Unrestrained Vehicles

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REFERENCE 1 presented an analysis of an unrestrained vehicle that is in flight and in quasi-static equilibrium by virtue of its inertial forces. A singularity appeared in the formulation that was identified as a physical aeroelastic divergence. This is not the case. Rather than a divergence, it is only a singularity that appears at the dynamic pressure at which the principal axis (i.e., the mean longitudinal reference

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